A Dynamic Approach To Conditionals

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I will focus on the role of logical dynamics in understanding and modeling several types of "conditionals" such as counterfactuals, doxastic conditionals and quantum conditionals. My aim is to provide a comparative analysis of these conditionals, centered around the idea of "action-based reasoning". I propose a reinterpretation of these various types of conditionals in a common setting, in terms of "dynamic conditionals" with a semantics based on "dynamic frames".

This approach has a number of advantages: (1) it clarifies some common misconceptions, highlighting the differences and similarities between quantum conditionals, doxastic conditionals and counterfactuals; (2) it shows how to generalize the logic of conditionals presented in [7], and (3) it provides a dynamic understanding of counterfactuals, which ties in with D. Lewis' spheres-semantics in [8].

The unifying formal setting that I propose is given by "dynamic frames": these provide the natural generalized models for the language of PDL (Propositional Dynamic Logic), being essentially labelled Kripke frames in which some of the basic relations are labelled by "propositions" (subsets of the state space). The semantics of classical PDL corresponds to interpreting these relations R^P as classical "test" actions (that do not change the input-state, but only test if it satisfies the given proposition P). In belief-revision contexts, these relations can have at least two different (but related) interpretations: (a) a "static" one, in terms of "conditional beliefs": the agent believes Q conditioned by P; (b) a "dynamic" one, in terms of "beliefs that might be held after obtaining new information": after learning P (and correspondingly revising his beliefs), the agent believes Q. In quantum logic, the same type of relations can be given a dynamic interpretation, in terms of "quantum tests" (yes-no measurements). This throws a new light on the usual "quantum implication" (the so-called Sasaki hook), which can now be understood as a dynamic modality: after a successful quantum test of (a testable property) P, the system satisfies Q. Finally, in the context of counterfactuals, these relations can also be given a dynamic interpretation, in terms of "small changes" or small variations of (the initial conditions of) the real world: events that "could *easily* have happened". In this interpretation, the meaning of the counterfactual conditional is that: either there is no accessible world that satisfies P, or Q holds at the first P-worlds that one encounters by repeatedly applying discrete "small changes" to the real world. These "small changes" comprise both actions that could actually have been performed in physical reality in the past, present or future, as well as Lewis's "small miracles" (in [9]) seen as violations of the laws of nature. This dynamic approach is a special case of D. Lewis' semantics of counterfactuals, in which the selection function is generated by a given set of concrete "small changes". At a conceptual level, our dynamic logical approach to counterfactuals bears some similarity to the so-called "structural account of counterfactuals", e.g. J. Pearl's "mini-surgeries" or "interventions", representing minimal changes to a (causal) model (see [1] for a recent overview).

I compare these various notions of dynamic conditionals, in terms of the different modal laws they satisfy and their corresponding frame conditions. I show that these laws encode different types of dynamics, presenting both the differences and some surprising similarities between the belief-revision conditional, counterfactuals and quantum conditionals.

While this dynamic logic approach is new in dealing with counterfactuals, it has already proven its merit in dealing with the mentioned other types of conditionals. For a dynamic logic setting that deals with quantum conditionals, I refer to my joint work on dynamic quantum logic [2,3]. Similarly, recent work on Dynamic Epistemic Logic models the use of doxastic conditionals in the context of belief revision and belief update [4,5,6].

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[1] H. Arlo-Costa, "The Logic of Conditionals", *Stanford Encyclopedia of Philosophy*, entry Sept. 2007.
[2] A. Baltag and S. Smets, "Complete Axiomatizations Of Quantum Actions", *International journal of theoretical physics*, 44(12), p.2267-2282, 2005
[3] A. Baltag and S. Smets: "LQP: The Dynamic Logic of Quantum Information", in *Mathematical Structures in Computer Science,* Special Issue on Quantum Programming Languages, 16(3): p.491-525, 2006.
[4] A. Baltag and S. Smets, "Conditional Doxastic Models: A Qualitative Approach to Dynamic Belief Revision", *Electronic Notes in Theoretical Computer Science*, 165, pp. 5-21, 2006.
[5] A. Baltag and S. Smets, "A Qualitative Theory of Dynamic Interactive Belief Revision", to appear in G. Bonanno, W. van de Hoek, M. Woolridge (eds.) *Texts in Logic and Games*, Amsterdam University Press.
[6] J. van Benthem, "Dynamic Logic for Belief Revision", ILLC Tech Report, University of Amsterdam, 2006. To appear *in Journal of Applied Non-Classical Logics.*
[7] B. Chellas, "Basic Conditional Logic", *Journal of Philosophical Logic*, 4:2, pp.133-153 , 1975.
[8] D. Lewis, *Counterfactuals*, Blackwell Publishing, 1973.
[9] D. Lewis, "Counterfactual Dependence and Time's Arrow", *Noûs*, 13(4), pp. 455-476, 1979.